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DHH-003-016202 Seat No. ________

M. Sc. (Maths) (Sem. II) (CBCS) Examination

April / May – 2015

CMT-2002 : Complex Analysis

Faculty Code : 003
Subject Code : 016202

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Answer all questions.
(ii) Each question carries 14 marks.
(iii) The figures on the right indicate the marks alloted to the question.

1 Answer any seven. Choose the correct answer. **2x7=14**

(1) The inverse of a bilinear transformation $\frac{az+b}{cz+d}$ is _____.

(A) $\frac{cz+d}{az+b}$ (B) $\frac{az-b}{-cz+d}$

(C) $\frac{dz-b}{-cz+a}$ (D) $\frac{dz+b}{cz+a}$

(2) If γ is a circle with centre 1 and radius 2 then

$\int_{\gamma}\frac{dz}{z-1} =$ _______.

(A) 1 (B) $2\pi i$

(C) 2 (D) $4\pi i$

(3) If $\gamma(t) = -1+2e^{6\pi it}$, $\forall t \in [0,1]$ then $n(\gamma, 0) =$_____.

(A) –1 (B) 3

(C) –3 (D) 6

(4) The right side of the line $y = x$ w.r.t. the orientation $(1+i, 0, -1-i)$ is _______.

(A) $\{z \in \mathbb{C} | \, img\, z > \text{Re}\, z\}$ (B) $\{z \in \mathbb{C} | \, img\, z < \text{Re}\, z\}$

(C) $\{z \in \mathbb{C} | \, \text{Re}\, z > 0\}$ (D) $\{z \in \mathbb{C} | \, img\, z > 0\}$

(5) ________ are linearly independent solutions of $y''-3y'+2=0$ on $\mathbb{R}$.

(A) $e^{t}, t e^{t}$ (B) e^{t}, e^{2t}

(C) $e^{2t}, t e^{2t}$ (D) e^{-t}, e^{-2t}

(6) _______ is a true statement.

(A) every piecewise smooth path is rectifiable

(B) every bilinear transformation has two fixed points

(C) every analytic function is an open map

(D) every continuous complex function has a primitive

(7) 0 is an essential singularity of ______.

(A) $\cos\left(e^{\frac{1}{z}}\right)$ (B) $\cos z$

(C) $\frac{\sin z}{z}$ (D) $\frac{1}{z}$

(8) If "a" is a pole of order n of f and $m \in \mathbb{N}$ then "a" is a pole of order _____ of f^m.

(A) m (B) n

(C) mn (D) $\frac{n}{m}$

(9) $3z^7 + 5z - 1$ has _______ zero/zeros in $|z| < 2$.

(A) 1 (B) 5

(C) 7 (D) 3

(10) If $f : D \to D$ is analytic and $f(0) = 0$ then _____

(A) $\left|f^1(0)\right| > 1$ (B) $\left|f\left(\frac{1}{2}\right)\right| \le \frac{1}{2}$

(C) $\left|f(1+i)\right| > \sqrt{2}$ (D) $f^1(0) = 2$

2 Answer any two : **2x7=14**

(a) Define total variation $V(\gamma)$ of a function $\gamma : [a,b] \to \not\subset$ of bdd variation. State, without proof, the formula to find $V(\gamma)$ of a piecewise smooth path $\gamma : [a,b] \to \not\subset$ and find $V(\gamma)$ of the line segment $\nu : [0,1] \to \not\subset$ joining two distinct points $z, w \in \not\subset$.

(b) State and prove symmetry principle of bilinear transformations.

(c) Find the bilinear transformation taking $i \to 1, 0 \to \infty$ and $-i \to 0$.

3 (a) Define winding number $n(\gamma, a)$ of a closed rectifiable path ν w.r.t. $a \notin \{\gamma\}$. Prove that $n(\gamma, \cdot)$ is constant on any component of $\not\subset \setminus \{\nu\}$. **7**

(b) State and prove that fundamental theorem of calculus for line integrals. **7**

OR

3 (a) If $\gamma : [a,b] \to \not\subset$ is a rectifiable path and $f : \{\gamma\} \to \not\subset$ is continuous then prove that **7**

$$\left|\int_\gamma f\right| \le \int_\gamma |f||dz| \le V(\gamma) \cdot \sup\left|f(0)\right| z \in \{\nu\}.$$

(b) State and prove maximum modulus theorem for analytic functions on a region. **7**

4 Answer any two : **2x7=14**

(a) Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi, \forall z \in \not\subset, |z| < 1$.

(b) If $f : G \to \not\subset$ is analytic and $\overline{B}(a, \sim) \subset G$ then write down the formula, without proof, for $f^n(a)$.

Evaluate $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$, where $n \in N$ and $\gamma(t) = e^{it}, \forall t \in [0, 2\pi]$.

(c) State, without proof, Cauchy's theorem (first version) and Cauchy's integral formula (second version).

Prove that Cauchy's theorem (first version) implies Cauchy's integral formula (second version).

5 Answer any two : **2x7=14**

(a) State, without proof, open mapping theorem for analytic functions. If $G \subset \not\subset$ is a region and and $f : G \to \not\subset$ is one-one and analytic then prove that $f^{-1} : f(G) \to \not\subset$ is analytic and $\left(f^{-1}\right)'(w) = \frac{1}{f^1\left(f^{-1}(w)\right)}, \forall w \in f(G)$.

(b) State, without proof, Laurent's theorem. Find the Laurent's expansion of $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in $|<|z| < 3$.

(c) Show that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$.

(d) State, without proof, Rouche's theorem. How many zeros does $z^4 - 3z + 1$ have in the open unit disc ? Prove your answer.
